Design of Photonic Crystals Devices with Defects

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Abstract **- A powerful and efficient method based on the leaky mode propagation method and recently applied by the authors to model defect-free optical periodic structures, is used to characterize photonic bandgap structures incorporating multiple defects, having arbitrary shape and dimensions. The importance of the defect-mode characterization in photonic bandgap materials materials is due to the intensive use of defects for light localization to design very promising optical devices. In order to prove the usefulness of the method, design of an optical filters for dense wavelength division multipexing applications, has been carried out by the developed model.**

I. INTRODUCTION

Waveguiding Photonic Band-Gap (WPBG) structures, i.e. regular periodic structures in which a transversal resonant condition is created for the light, and Fully etched WPBG (FWPBG, in which the slots are etched down to the substrate) are very promising for their applications in optoelectronics, particularly for ultracompact photonic microcircuits, low threshold, high efficiency light emitting devices [1] and high performance optical resonators and filters for Dense Wavelength Division Multiplexing (DWDM) communication systems [2]. In fact, it is well known the capability of PBG-based devices to control the light [3], providing either the inhibition or localization of the radiation. Optimal design of such devices requires the use of appropriate numerical models. The Leaky Mode Propagation method (LMP) has been recently used by the authors to develop a very powerful model of both infinitely long and finite-size, defect-free, PBG structures [4-5].

II. MODEL DESCRIPTION

The LMP method is faster than other numerical methods, does not require any analytical approximations and provides a good numerical stability. It allows to take into account all the physical phenomena occurring when a wave propagates inside a periodic structure as PBG one having deep grooves, high refractive index contrast, and finite length. Losses due to either a stopband (Bragg interaction) or power leakage caused by out of plane optical scattering or radiation are also taken into account.
The model has been demonstrated also to be capable to characterize the out-of-plane losses of a 2-D FWPBG structure by referring to a 1-D FWPBG model [5-6]. Results of intensive and accurate comparisons has accomplished by the authors among the most used numerical methods and the LMP one, can be found in Ref. 4,5. The model has been implemented in a very fast code in FORTRAN 77 language, running on a personal computer and able to provide all the parameter values in a few seconds: mode propagation constants, harmonics and total field distribution, transmission and reflection coefficients, Poynting vector, forward and backward power flow, guided power and total losses.
Our model enables the designer to have a complete view over the physical and geometrical device features, and to draw very easily optimal design rule of PBG-based devices.
Anyway the principle of operation of PBG-based devices generally implies the localization of light by introducing a defect in the structure, i.e. a region in which the periodicity of the refractive index is interrupted. Therefore, a useful and complete model must be able to characterize also defect modes. Modeling the presence of an arbitrary number of defects, plugged in the regular periodic region, having any arbitrary shape and dimensions, is the aim of this paper.
To this aim, we have modeled a periodic structure with defects as an array of devices without defects, linked each other by pieces of slab waveguides, as sketched in Fig.1. In this way, we develop a model that is general, able to characterize any complex PBG structure, composed of an arbitrary number M of partially or fully etched gratings separated by M-1 defects. The defects can be different each other in terms of extension and thickness.
Referring to Fig.1 we denote the generic grating of the array, posed in the h position (h = 1, M), as h-grating, which length is L_h. Between the h-1 grating and the h-grating is plugged the h-defect, which length is d_h and propagation constant β_h. For each regular periodic grating we assume: a)both sinusoidal and generally trapezoidal profile, having period Λ_h; particular cases of the trapezoidal shape of the etched region are the triangular, rectangular and saw-tooth profiles; b) isotropic and homogeneous unperturbed layers (cover, substrate); c) a finite length along the z propagation direction and infinite length along the y direction.
At first, each grating is singularly considered and characterized assuming it out of the array and infinitely long. Transverse field solution F_{PBG} (E_y for TE and H_y for TM polarization) of the scalar wave equation in the perturbed region is the following:

$$F_{PBG}(x,z) = \sum_n f_n(x)\exp(jk_{zn}z) \quad 0 \le x \le t_g$$

where $f_n(x)$ is the amplitude function of the n-th Bloch harmonic. It is a function of the depth x and is evaluable as detailed in author's Ref. 4. Due to the Bloch-Floquet phase relationship the following condition applies:

$$k_{zn} = \beta_0 + j\alpha + \frac{2n\pi}{\Lambda} = \beta_n + j\alpha$$

where the leakage factor α (> 0) takes into account Bragg reflection and power radiation.

The detailed characterization procedure and the complete theory and model expressions can be found in author's Ref.4. Then, we account for the finite-length of each grating by calculating the reflection and transmission coefficients, R_p and T_p, respectively, by using a solution, F, of the transverse field, which is a linear combination of two linearly independent solutions $F^{(a)}$ and $F^{(b)}$ of the infinitely long one, obtained in the first step.

We have: $F = a*F^{(a)} + b*F^{(b)}$ where $F^{(a)}$ is the "forward" solution, obtained by solving the infinitely-long grating seen in the +z direction, and $F^{(b)}$ is the backward one, obtained by solving the infinitely-long grating seen in the –z direction.

Obviously, if the profile of the etched slots is symmetric with respect to the x-axis the amplitude functions $f_n^+(x)$ and $f_n^-(x)$ are identical. a and b are arbitrary coefficients to be determined.

Imposing the reflected and transmitted field continuity conditions at the input/output sections of the single grating a linear system is provided in four unknowns: a, b, ρ and τ, detailed in Ref. 4, 5. ρ and τ are the field reflection and trasmission coefficients, respectively. The system can be analytically solved, allowing the power reflectivity $R_P = |\rho|^2$ and the power transmittivity $T_P = |\tau|^2$ and then the out of plane losses $L_P = 1 - R_P - T_P$ to be determined.

At this stage, we are able to model also PBG devices with defects.

We refer the model expressions relevant to the most general situation in which the etching profile of each grating is asymmetrical and the defects have different thickness.

An inspection about the waves traveling inside the structure shows that at the input end of the generic h-grating of the array, impinges the field transmitted from the (h-1)-grating and the field traveling into the (h-1)-defect, which length is $d_{(h-1)}$. This field is due to the multiple reflections occurring between the output end of the (h-1)-grating and the input end of the h-grating. The field propagating into the (h-1)-defect suffers from the phase shift due to the defect length $d_{(h-1)}$. Moreover, at the output section of the h-grating impinges the field reflected by the input section of the (h+1)-grating and the field traveling into the h-defect, which suffers from the phase shift due to the distance d_h.

We define as:

$$\rho_h = \frac{\varphi^{(h)}_{rif}}{\varphi^{(h)}_{inc}} \quad \text{and} \quad \tau_h = \frac{\varphi^{(h)}_{tr}}{\varphi^{(h)}_{inc}}$$

the coefficients accounting for the field reflected and transmitted by the h-grating, respectively (see Fig.1). Then, the amplitudes of the beams impinging and reflected at the input section of the h-grating, $\varphi^{(h)}_{inc}$ and $\varphi^{(h)}_{rif}$, can be written as follows:

$$\varphi^{(h)}_{inc} = t_{h-1} F_s^{(h-1)}(x) \exp(j\beta_u d_0) \exp(j\sum_{i=1}^{h-1} \beta_i d_i)$$

$$\varphi^{(h)}_{rif} = \rho_h t_{h-1} F_s^{(h-1)}(x) \exp(j\beta_u d_0) \exp(j\sum_{i=1}^{h-1} \beta_i d_i)$$

Then, the amplitudes of the fields incident and transmitted at the output section of the h-grating, $\psi^{(h)}_{inc}$ and $\varphi^{(h)}_{tr}$, can be written as follows:

$$\psi^{(h)}_{inc} = \rho_{h+1} t_h F_s^{(h)}(x) \exp(j\beta_u d_0) \exp(j\sum_{i=1}^{h-1} \beta_i d_i) \exp(j2\beta_h d_h)$$

$$\varphi^{(h)}_{tr} = \tau_h t_{h-1} F_s^{(h)}(x) \exp(j\beta_u d_0) \exp(j\sum_{i=1}^{h-1} \beta_i d_i)$$

being $\psi^{(h)}_{inc}$ the field amplitude reflected by h+1 grating and incident at the output section of the h grating; having defined: $t_{h-1} = \prod_{i=1}^{h-1} \tau_i$ and being $F^{(h)}_{inc} = F_s^{(h)}(x) \exp(j\beta_h d_h)$ and $F_{inc} = F_s(x) \exp(j\beta_u d_u)$ the fields impinging at the input section of the h-grating and of the first grating of the array, and, then, propagating in the h-slab and in the input slab, respectively. The propagation constant β_u is the same for the input/output slabs; β_h is the propagation constant of the h-defect (slab). $F_s^{(h)}(x)$ and $F_s(x)$ are the amplitude functions of the fields previously defined.

By imposing the appropriate field continuity conditions at the input and output section of each grating of the array, we obtain M systems, each having four unknowns: a_h, b_h, ρ_h, τ_h being h = 1, 2, …M.

We have:

$$\tau_h = \frac{H'_h (C_h F'_h - D'_h E_h)}{H''_h (A_h F'_h - B'_h E_h \exp(2\delta L_h) + [B'_h C_h \exp(2\delta L_h) - A_h D'_h] \rho_{h+1} \exp(j2\beta_h d_h))} \exp(\delta L_h)$$

$$\rho_h = \frac{B_h F'_h - A'_h E_h \exp(2\delta L_h) + [A'_h C_h \exp(2\delta L_h) - B_h D'_h] \rho_{h+1} \exp(j2\beta_h d_h)}{A_h F'_h - B'_h E_h \exp(2\delta L_h) + [B'_h C_h \exp(2\delta L_h) - A_h D'_h] \rho_{h+1} \exp(j2\beta_h d_h)}$$

$$a_h = \frac{H'_h (F'_h - D'_h \rho_{h+1} \exp(j2\beta_h d_h))}{[A_h F'_h - B'_h E_h \exp(2\delta L_h)] + [B'_h C_h \exp(2\delta L_h) - A_h D'_h] \rho_{h+1} \exp(j2\beta_h d_h)}$$

$$b_h = a_h \frac{C_h \rho_{h+1} \exp(j2\beta_u d_h) - E_h}{F'_h - D'_h \rho_{h+1} \exp(j2\beta_h d_h)} \exp(2\delta L_h)$$

where a_h e b_h are the a and b constants[4,5] relevant to the h-grating, $\delta = -\alpha + j\beta_0$.

Moreover:

$$A_h = \sum_{n=-p}^{+\infty} \left[\int_{-\infty}^{+\infty} f_n(x) F_s^{*(h-1)}(x) dx \right]$$

$$A'_h = \sum_{n=-p}^{+\infty} \left[\int_{-\infty}^{+\infty} f_n^-(x) F_s^{*(h-1)}(x) dx \right]$$

$$B_h = \sum_{n=-\infty}^{-(p+1)} \left[\int_{-\infty}^{+\infty} f_n(x) F_s^{*(h-1)}(x)dx \right]$$

$$B'_h = \sum_{n=-\infty}^{-(p+1)} \left[\int_{-\infty}^{+\infty} f_n^-(x) F_s^{*(h-1)}(x)dx \right]$$

$$C_h = \sum_{n=-p}^{+\infty} \exp(jnK_h L_h) \left[\int_{-\infty}^{+\infty} f_n(x) F_s^{*(h-1)}(x)dx \right]$$

$$D'_h = \sum_{n=-\infty}^{-(p+1)} \exp(-jnK_h L_h) \left[\int_{-\infty}^{+\infty} f_n^-(x) F_s^{*(h-1)}(x)dx \right]$$

$$E_h = \sum_{n=-\infty}^{-(p+1)} \exp(jnK_h L_h) \left[\int_{-\infty}^{+\infty} f_n(x) F_s^{*(h-1)}(x)dx \right]$$

$$F'_h = \sum_{n=-p}^{+\infty} \exp(-jnK_h L_h) \left[\int_{-\infty}^{+\infty} f_n^-(x) F_s^{*(h-1)}(x)dx \right]$$

$$H'_h = t_{h-1} \exp(j\sum_{i=0}^{h-1} \beta_i d_i) \left[\int_{-\infty}^{+\infty} F_s^{(h-1)}(x) F_s^{*(h-1)}(x)dx \right]$$

$$H''_h = t_{h-1} \exp(j\sum_{i=0}^{h-1} \beta_i d_i) \left[\int_{-\infty}^{+\infty} F_s^{(h)}(x) F_s^{*(h-1)}(x)dx \right]$$

where p is the highest order backward harmonic (see Ref. 5) and $K_h = 2\pi/\Lambda_h$.

The solution for the whole structure starts from the last grating of the array (h = M and $\rho_{M+1} = 0$) and goes back towards the (M-1)-grating and so on, to the first one.

Finally, the field reflection and transmission coefficients of the whole defective PBG, ρ_d and τ_d, respectively, and, then, the power reflectance and transmittance, R_p and T_p, and the out-of-plane losses L_p can be determined.

Infact, we have:

$$\rho_d = \rho_1 \quad \text{and} \quad \tau_d = \left[\prod_{i=1}^{M} \tau_i \right] * \exp(j\sum_{i=1}^{M-1} \beta_i d_i) ;$$

then:

$R_P = |\rho_1|^2$; $T_P = |\tau_d|^2$ and $L_P = 1 - R_P - T_P$.

III. RESULTS

By the developed model, two DWDM filters have been designed, whose performances advance the state of the art of commercial DWDM optical filters. The aim is to show the usefulness and capabilities of the model to design optimization.

The technologies chosen are $GaAs/Al_xO_y$ and Si/SiO_2, due to their reliability and usefulness to optoelectronic monolithic integration. The chosen operating wavelength is $\lambda = 1.55$ µm.

Filter # 1 - The device structure is as in Fig.1. The design procedure starts from the determination of the parameters relevant to the defect-free device.

As a second step we plug in a $\lambda/4$ long defect at the center of the structure, which breaks off the regular periodicity and splits the grating into two equals half-length gratings, as in Fig.1. The length of the defect has been appropriately chosen to have a constructive interaction between the counterpropagating beams in the defective region, in correspondence of the operating wavelength $\lambda = 1.55$ µm.

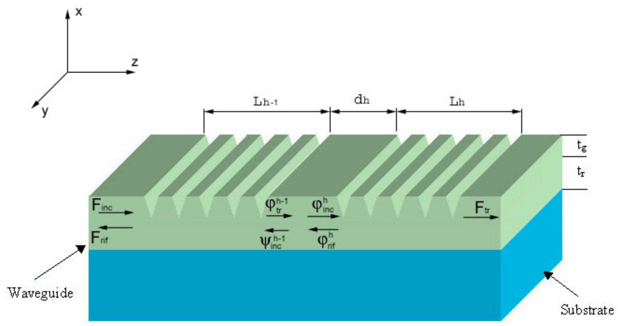

Fig.1. Model of 1-D WPBG device with defect.

This gives rise to a field concentration in the defect region whilst the field vanishes in the periodic part of the device because the operating wavelength is in the bandgap of the grating where the propagation is prohibited. It results a filtering effect in a very narrow band around $\lambda = 1.55$ µm, as we will see later. The filter is modeled as a couple of gratings each having length $L_s = 40 \cdot \Lambda$ connected by a single defect, i.e. a piece of slab having $d_1 = 0.1364$ µm. The value of d_1 has been chosen to have a peak in the transmittivity spectrum in correspondence of $\lambda = 1.55$ µm. Fig. 2 shows the transmittivity spectrum relevant to the filter; the presence of an allowed state in the bandgap, i.e. of a transmission peak at $\lambda = 1.55$ µm, confirms the validity of our calculations.

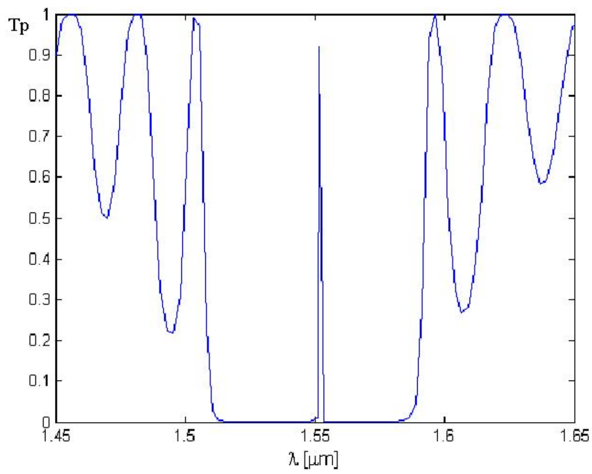

Fig. 2. Dispersion curve of transmittivity T_p relevant to the filter # 1 having a $\lambda/4$ defect.

The designed filter has a total length of 21.96 µm and a bandwidth, calculated at –3 dB, equal to 0.16 nm (20 GHz).

In Fig.3 the very good confinement of the field Ey for TE modes in the defective region (or cavity) is shown.

The high value of the quality factor Q is related to the field confinement.

By perfoming a lot of simulation we have found also that a reduction in the grating length makes the transmittivity peak T_{max} higher and the out-of-plane losses lower, but the channell-width $\Delta\lambda$ also enlarges.

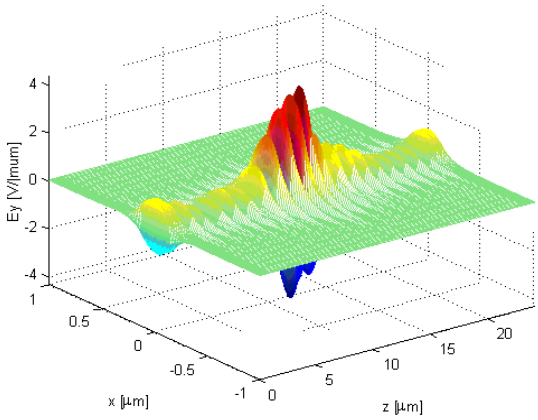

Fig. 3. Total field propagation in the filter #1 at resonance.

This last parameter depends also by the field confinement into the cavity. Both T_{max} and $\Delta\lambda$ can be improved by lowering the refratcive index of the substrate and by a deeper etching of the periodic region. The first solution allows a better field confinement thus reducing the evaneshent field in the substrate; the second one allows to obtain a better reflectivity by reducing the out-of-plane losses. Then, we can optimize the transmittivity maintaining the length as small as possible.

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